



## THE LARGE-SCALE PECULIAR VELOCITY FIELD IN FLAT MODELS OF THE UNIVERSE

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### *Abstract*

The inflationary Universe scenario predicts a flat Universe and both adiabatic and isocurvature primordial density perturbations with the Zel'dovich spectrum. The two simplest realizations, models dominated by hot or cold dark matter, seem to be in conflict with observations. We examine flat models with two components of mass density, where one of the components of mass density is smoothly distributed, and compute the large-scale ( $\gtrsim 10h^{-1}Mpc$ ) peculiar velocity field for these models. For the smooth component we consider relativistic particles, a relic cosmological term, and light strings. At present the observational situation is unsettled; but, in principle, the large-scale peculiar velocity field is a very powerful discriminator between these different models.



## I. INTRODUCTION

Studies of the very early history of the Universe ( $t \ll 0.01s$ ) have resulted in a number of very definite predictions for the initial data appropriate for structure formation in the Universe. The inflationary scenario (Guth, 1981; Linde, 1982; Albrecht and Steinhardt, 1982; for a recent review, see Turner, 1986a) predicts primordial density perturbations, both adiabatic (Hawking, 1982; Starobinskii, 1982; Guth and Pi, 1982; Bardeen, Steinhardt, and Turner, 1983) and in the case of an axion-dominated Universe, isocurvature also (Steinhardt and Turner, 1983; Linde, 1985; Seckel and Turner, 1985), with the Zel'dovich spectrum, and that the Universe today should be indistinguishable from the flat, Einstein-deSitter model. Primordial nucleosynthesis constrains the fraction of critical density in baryons ( $\equiv \Omega_b$ ) to be less than  $0.035h^{-2}$  (Yang et al. 1984), strongly suggesting that most of the mass density in the Universe is non-baryonic. [Here and throughout the present value of the Hubble parameter is taken to be,  $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ .] There is a long list of candidate relic particles that might be left over from the earliest moments of the Universe and whose predicted relic abundance is such that they could be the dark component (for a recent review of the dark matter candidates, see Turner, 1986b). Fortunately, such relics can be lumped into two limiting classes: hot (so far only neutrinos) and cold (essentially all the other candidates). These highly specific suggestions for the initial data appropriate for the structure formation problem lead to three well-defined scenarios: hot (adiabatic) and cold (adiabatic and isocurvature), neither of which results in a totally satisfactory scenario of structure formation.

Because the observational data seem to indicate that the fraction of critical density which is contributed by matter which clusters on scales less than about  $10h^{-1} \text{ Mpc}$  is

only about 0.1 to 0.3, somewhat more speculative scenarios have put forth where the yet undetected and unaccounted for 70% – 90% of the critical density is in the form of a smoothly-distributed component. Suggestions for the smooth component include 'failed galaxies' (Kaiser, 1986; Bardeen et al., 1986), relativistic particles produced by the recent (redshift of decay  $z_d \leq 10$ ) decay of unstable relic particles (Dicus, Kolb, and Teplitz, 1977; Turner, Steigman, and Krauss, 1984; Gelmini, Schramm, and Valle 1984; Olive, Seckel, and Vishniac, 1985), a relic cosmological term of unexplained origin (Turner, Steigman, and Krauss, 1984; Peebles, 1984), or even fast-moving, light cosmic strings (Vilenkin, 1984; however, also see Turner, 1985a).

We have at our disposal a number of observational probes which can be used to discriminate between these different flat models all of which are motivated by early Universe physics. They include the angular structure of the cosmic microwave background (CMB) (see, e.g., Vittorio and Silk, 1984; Bond and Efstathiou, 1984; Bonometto, Lucchin, and Valdarnini, 1984), the distribution of mass in the Universe (as quantified by the galaxy-galaxy correlation function, the cluster-cluster correlation function, the existence of voids, superclusters, etc.), the usual cosmological tests (magnitude-redshift diagram, etc.; Charlton and Turner, 1986), and the peculiar velocity field of the Universe (see, e.g., Kaiser, 1983; Vittorio and Silk, 1985a). In this paper we focus on the large scale peculiar velocities predicted in these various scenarios.

In the linear regime, peculiar velocities are related directly to the density perturbations which gave rise to them via gravitational forces. As such they provide a very direct probe of the density field of the Universe on large scales, where the perturbations are still linear. This is in contrast to the distribution of visible galaxies which only provides a direct

probe of the density field of the Universe if bright galaxies are a good tracer of the mass distribution.

In this paper we report the results of model calculations for the expected peculiar velocity field for all the models discussed above. The observational data which are relevant for comparison are the dipole anisotropy of the microwave background radiation and the anisotropy of the Hubble flow on various scales. At present, the observational situation with respect to the anisotropy of the Hubble flow is still in a state of flux and far from being clear. Although the quantity and quality of observations have increased markedly recently, a number of seemingly contradictory observations exist at present. This precludes a definitive comparison of our results with observations of the anisotropy of the Hubble flow. This is unfortunate because the observational data could be used to strongly constrain the various scenarios and perhaps even rule out some models. Given the observational situation at this time, we will simply present our results as 'theoretical predictions.'

The outline of the rest of the paper is as follows. In the next section we review and discuss the observational data and indicate how the data along with our results can be used to test the various models. In Sec. III, we discuss the models and our method of computing the large-scale peculiar velocity field, and in Sec. IV we present our results in the form of analytic fits to the numerical results. We conclude with a brief summary.

## II. OBSERVATIONS

The CMB dipole anisotropy ( $\delta T = 3.5 \pm 0.1 mK$ ) is indicative of our peculiar motion and implies a Local Group velocity  $v_{LG} = 610 \pm 50 km s^{-1}$ , relative to the CMB, in a direction which is about  $45^\circ$  away from the Virgo Cluster (for a recent review see Lubin and Vilella, 1986). This cluster dominates the local dynamics. The Local Group is falling

into Virgo with a velocity which is generally accepted to be  $v_{inf} = 250 \pm 50 km s^{-1}$  (see, e.g., Yahil, 1985). By subtracting  $\vec{v}_{inf}$  from  $\vec{v}_{LG}$ , one can attempt to evaluate the velocity of the Virgo cluster as a whole relative to the CMB. The velocity obtained in this way is  $v_{VC} = 470 \pm 70 km s^{-1}$ , in the general direction of the Hydra-Centaurus Supercluster, an association of Centaurus, Antila, Hydra and several other smaller clusters at a mean distance of  $\sim 30 h^{-1}$  Mpc. Whether or not the Hydra-Centaurus Supercluster is responsible for  $v_{VC}$  is still not clear. Regardless, from this simple analysis one concludes that the Local Group velocity relative to the CMB arises as the combined effect of the infall into Virgo and of the motion of the Virgo Cluster as a whole. We will compare the motion of the Virgo Cluster as a whole with respect to the CMB (a so-called "corrected" dipole anisotropy),  $v_{VC}$ , to our model predictions for the dipole anisotropy.

Determining how large a volume one must consider so that the matter within this volume is at rest with respect to the CMB is crucial for comparing the observations with theory. This is a point that we shall emphasize again in Sec. IV. An ingenious method of clarifying the locality of the CMB dipole anisotropy involves comparing the measured velocity of the Local Group relative to a given sample of galaxies, selected in a volume big enough not to be strongly affected by local nonlinearity. If these galaxies are unperturbed tracers of the Hubble flow, the peculiar velocity of the Local Group relative to the sample should be equal to the velocity of the Local Group relative to the CMB. In other words, the matter in the considered volume and the CMB are at rest and the dipole anisotropy observed by us is due to local effects. If the two velocities are different, a coherent motion relative to the CMB of all the matter inside the volume is implied.

The IRAS Point Source Catalogue provides for the first time a galaxy sample uniformly

selected and with a nearly complete sky coverage. The analysis of this sample has proven to be a powerful probe of the Hubble flow field. It has been shown that the distribution of galaxies in the catalogue exhibits a dipole anisotropy in reasonable agreement with the direction of the CMB dipole anisotropy (to within  $\sim 20^\circ - 30^\circ$ ; Yahil et al., 1986; Meiksin and Davis, 1985). The deepness of the IRAS sample is estimated to be  $\lesssim 50h^{-1}$  Mpc, and, it is possible that the bulk of the anisotropy is generated on even smaller scales. This seems to suggest a very local origin of the CMB dipole anisotropy, implying that the peculiar velocity field on scales  $\gtrsim 50h^{-1}Mpc$  should be very small. This conclusion will be tested and better quantified with the acquisition of redshifts of the galaxies in sample.

The locality (or nonlocality) of the dipole anisotropy can also be studied by analyzing smaller galaxy samples, with available independent measurements of radial velocities and distances. Different results have appeared in the literature. Rubin et al. (1976) found that the Local Group is moving relative to a background sample of 96 spiral galaxies (mean redshift  $5100kms^{-1}$ ) with a velocity of  $450kms^{-1}$  in a direction which is almost orthogonal to that inferred from the CMB dipole anisotropy. Hart and Davies (1982) found that the velocity of the Local Group relative to a shell of 84 spiral galaxies is in good agreement (both in amplitude and direction) with the one inferred from the CMB. Their result implies that the velocity of a sphere of radius  $r \sim 25h^{-1}Mpc$  has a residual velocity relative to the CMB of only  $130 \pm 70kms^{-1}$ . deVaucouleurs and Peters (1984) measured the velocity of the Local Group relative to galaxy shells of different radii. They concluded that a shell of  $25h^{-1}Mpc$  radius has a peculiar velocity of  $\sim 350kms^{-1}$  and commented that on scales  $\gtrsim 40h^{-1}Mpc$  matter is at rest with respect to the CMB. Staveley-Smith and Davies (1985), after analyzing a sample of 200 galaxies, suggested that the bulk of

the dipole anisotropy is generated on scales  $\gtrsim 40Mpc$ . More recently, Collins et al. (1986) have reobserved half of the old Rubin et al. (1976) sample and, confirming the original result, concluded that a spherical volume of size  $\sim (50h^{-1}Mpc)^3$  has a peculiar velocity of  $970 \pm 300 km s^{-1}$  relative to the CMB.

Aaronson et al. (1985) measured the peculiar velocity of the Local Group relative to a set of 11 clusters at distances of  $40 - 100 h^{-1}Mpc$ . They find a dipole anisotropy of the Hubble flow consistent both in direction and amplitude ( $780 \pm 188 km s^{-1}$ ) with the CMB; one would tentatively conclude that these clusters define quite accurately the comoving frame. The advantage of using clusters instead of galaxies rests on the greater accuracy in distance determinations.

Studying a sample of  $\sim 400$  elliptical galaxies, Burnstein et al. (1986) concluded that a volume of size  $\sim (60h^{-1}Mpc)^3$  is moving relative to the CMB with a velocity of  $600 km s^{-1}$ . Interestingly, they comment on the fact that their data reproduces both the Collins et al.(1986) and the Aaronson et al. (1986) results when similar subsamples are taken from their data.

### III. METHOD OF CALCULATION

#### *a) The Models*

Motivated by the inflationary Universe (and other theoretical prejudices; see, e.g., Dicke and Peebles, 1979) we are interested in flat models of the Universe (more precisely, curvature signature  $k = 0$ ). The Friedmann equation for these models takes a very simple form

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad (1)$$

where  $a(t)$  is the cosmic scale factor and  $\rho$  is the total energy density, which includes

matter, radiation, vacuum energy (equivalently, a cosmological term), and light strings. We will denote the fraction of critical density in each component today by  $\Omega_i$ .

All of the models have a component of matter which is non-relativistic and can clump; this component includes baryons and non-relativistic (NR) dark matter. It is this component which most of the cosmological observations determine and which constitutes only  $\sim 20\%$  of the critical density. We denote it by  $\Omega_{NR}$ .

The simplest models are those where  $\Omega_{NR}$  is equal to 1. That is, models where there is but one component to the energy density. The bulk of the mass density must of course be in the dark component, either hot or cold relics.

Next are the models where  $\Omega_{NR}$  is less than 1, and a smooth component contributes  $\Omega_{SM} = 1 - \Omega_{NR}$ . We consider the following possibilities for the smooth component: (i) Relic cosmological term: At present there is no fundamental understanding of why the cosmological term  $\Lambda$  is at least 122 orders of magnitude smaller than the only natural scale for it,  $c^5/(G\hbar)$ . In spite of this fact, we can at least entertain the possibility that there could be a small relic cosmological term today which provides  $\Omega_{SM}$ . In this model we shall further assume that the NR component behaves like cold dark matter, as Vittorio and Silk (1985b) and Turner et al. (1984) have pointed out that if the NR component is baryons or HDM excessive small-scale microwave anisotropy results.

(ii) Decaying Particles: If the smooth component is relativistic particles, then they must have been produced by the recent decay of a massive relic species, otherwise the Universe would never have been matter-dominated and density perturbations could never have grown. The kinematics of such a model Universe have been studied in detail by Turner (1985b). We will follow his treatment and refer the interested reader to his paper

for more details. The smooth component in this model is relativistic particles and today contributes  $\Omega_R = 1 - \Omega_{NR}$ . The decay epoch occurs at

$$(1 + z_D) \simeq \beta^{-1} \frac{\Omega_{NR}}{\Omega_R} \quad (2)$$

where  $\beta^{-1}$  is the ratio of the mass density of unstable massive particles (before they decay) to that in stable NR particles (baryons, etc.). The age of the Universe at the decay epoch is roughly equal to the lifetime of the decaying particle. [More precisely, the redshift when the age of the Universe is equal to the lifetime of the unstable particle is:  $1 + z(\tau) \simeq 0.8\beta^{-1}\Omega_{NR}/\Omega_R$ .]  $\Omega_R$  is related to the mass, lifetime and abundance of the decaying particle by

$$\Omega_R h^2 = 1.41(m_x/100eV)^{4/3}(n_x/n_\gamma)^{4/3}(\tau/10^9 yrs)^{2/3} \quad (3)$$

where  $n_x/n_\gamma$  is the pre-decay abundance of unstable particles relative to photons,  $m_x$  is the mass of the unstable particle, and  $\tau$  is its lifetime. The decaying particles themselves can either be hot or cold relics.

(iii) Light Strings: Vilenkin (1984; also see, Turner, 1985a) has suggested the possibility that the smooth component is either fast-moving strings or a network of light strings, either of which could have been produced in a relatively late phase transition ( $kT \simeq 10^4 GeV$ ). [These light strings are not to be confused with the heavy strings, responsible, in some scenarios, for the origin of density perturbations in the Universe; see, e.g., Zel'dovich (1979); Vilenkin (1985); Albrecht and Turok (1985); Turok (1985); Szalay and Schramm (1985).] Neither the fast-moving strings nor the network of strings can clump and so they too behave like a smooth component of energy density. The energy density in fast-moving strings decreases as  $1/(ta(t))$ , so that while the Universe is matter-dominated

$\rho_{string} \propto a(t)^{-5/2}$  and when the Universe is string-dominated  $\rho_{string} \propto a(t)^{-2}$  (just like a curvature-dominated model). On the other hand, the energy density in a string network always decreases as  $\rho_{string} \propto a(t)^{-2}$ . For these light string models we assume that the NR component behaves like CDM. If the NR component behaved like HDM or were baryons only, the predicted small-scale microwave anisotropy would be far too large (Vittorio and Turner, 1986).

(iv) Open model: For comparison purposes we also include a  $k < 0$  model, where  $\Omega_{NR} < 1$  and  $\Omega_{SM} = 0$ . As noted above it behaves like a flat model whose smooth component is a string network. As with the string models we will only consider the case where the NR component behaves like CDM.

#### *b) Density fluctuation spectrum*

It is generally assumed that the observed structure in the Universe is the result of the gravitational amplification of small, initial density fluctuations. Inflation provides definite predictions about the primordial perturbations—either adiabatic or isocurvature perturbations with the scale-invariant Zel’dovich spectrum. We will consider both possibilities in a CDM dominated universe. For hot dark matter (HDM) we consider only adiabatic fluctuations since they are indistinguishable from isocurvature fluctuations on the only relevant scales (those larger than the neutrino damping length) and isocurvature perturbations only arise in an axion-dominated Universe. For these spectra, perturbations on all length scales cross into the horizon with the same amplitude (equivalently, the power spectrum at very early times is:  $|\delta(k, t_i)|^2 \propto k$ ). However, those scales that enter the horizon before the epoch of matter-radiation equivalence ( $t \simeq 10^{10} \text{sec}$ ) cannot undergo significant growth until the Universe becomes matter-dominated. Those scales that enter the horizon

after matter domination undergo continuous growth. The period of interrupted growth for perturbations which enter the horizon before matter domination distorts the initially scale-invariant Zel'dovich spectrum. After that, density fluctuations on the scales of interest for the large scale structure are amplified irrespective of their wavenumber. Their common growth factor  $D(t)$  obeys the well known equation (see, e.g., Peebles, 1980):

$$\frac{1}{a^2} \frac{d}{dt} a^2 \frac{d}{dt} D = 4\pi G \rho_{NR} D \quad (4)$$

The functional form of  $D(t)$  will depend upon the particular cosmological model through  $a(t)$ .

As usual we decompose the density fluctuations in the NR component into their Fourier components,  $\delta(k, t)$ , labeled by comoving wavenumber  $k$ . We take the scale factor  $a(t)$  to be normalized so that comoving wavenumbers and wavelengths correspond to physical wavenumbers and wavelengths today (i.e.,  $a_{today} = 1$ ). After the matter-radiation equivalence epoch the evolution of  $\delta(k, t)$  factorizes:  $\delta(k, t) \equiv \delta_k D(t)$ .

In general  $\delta_k$  exhibits two characteristic length scales: a) the minimum scale  $L_D$  able to survive collisionless damping due to free-streaming, and b) the horizon length  $L_{EQ}$  at the matter-radiation equality. For models where the NR component is dominated by stable HDM,  $\delta_k$  is well fitted by:

$$|\delta_k|^2 = A k 10^{-2} \left(\frac{k}{k_\nu}\right)^{1.5} \quad (5)$$

where  $A$  is the overall normalization and  $k_\nu = 0.49 Mpc^{-1} \Omega_{NR} h^2$ . The spectrum exhibits only one scale, the neutrino damping scale  $L_D$  ( $\equiv 2\pi/k_\nu$ )  $\simeq 13(\Omega h^2)^{-1} Mpc \simeq L_{EQ}$  (Bond and Szalay, 1983). In the case of decaying HDM,  $k_\nu = 0.49 Mpc^{-1} \beta^{-1} \Omega_{NR} h^2$  and  $L_D = 13\beta(\Omega_{NR} h^2)^{-1} Mpc$ . The damping scale is reduced by  $(1 + z_D)$ , as the neutrino mass required increases with  $(1 + z_D)$ .

For models where the NR component is dominated by stable CDM and the primordial fluctuations are adiabatic, the spectrum has the following shape:

$$|\delta_k|^2 = A \frac{k}{[1 + \alpha k + \omega k^{1.5} + \gamma k^2]^2}. \quad (6)$$

where A is the overall normalization,  $\alpha = 1.7(\Omega_{NR} h^2)^{-1} Mpc$ ,  $\omega = 9.0(\Omega_{NR} h^2)^{-1.5} Mpc^{1.5}$ , and  $\gamma = 1.0(\Omega_{NR} h^2)^{-2} Mpc^2$  (see, e.g., Davis et al., 1985). For CDM,  $L_D$  is negligibly small. The slope of the spectrum changes from the primordial slope ( $\propto k$ ) to  $k^{-3}$  for wavenumbers  $\gg \frac{2\pi}{L_{EQ}}$ . This reflects the effect we discussed earlier, the fact that perturbations which enter the horizon before matter-radiation equivalence are essentially 'frozen in' until then. The dependence of the coefficients  $\alpha, \omega$ , and  $\gamma$  on  $\Omega_{NR} h^2$  reflects the dependence of these coefficients on the only scale in the problem,  $L_{EQ}$ . In a model dominated by unstable CDM the spectrum is also given by Eqn(6) and is obtained by substituting  $\Omega_{NR} \rightarrow \beta^{-1} \Omega_{NR}$ . As with unstable HDM, increasing  $\beta^{-1}$  (or equivalently,  $1 + z_D$ ) does not affect the shape of the spectrum, rather it merely shifts the spectrum to higher wavenumbers (the scales  $L_D$  and  $L_{EQ}$  are decreased).

For a model dominated by stable CDM with isocurvature fluctuations, the spectrum has the following shape:

$$|\delta_k|^2 = A \frac{k}{[1 + (\alpha k + \omega k^{1.5} + \gamma k^2)^{1.24}]^{1.61}}, \quad (7)$$

where A is the overall normalization,  $\alpha = 15.(\Omega_{NR} h^2)^{-1} Mpc$ ,  $\omega = 0.80(\Omega_{NR} h^2)^{-1.5} Mpc^{1.5}$ , and  $\gamma = 31.4(\Omega_{NR} h^2)^{-2} Mpc^2$  (Efstathiou and Bond, 1986). For a model dominated by unstable CDM with isocurvature perturbations the initial spectrum is also given by Eqn(7), by substituting  $\Omega_{NR} \rightarrow \beta^{-1} \Omega_{NR}$  as before.

### c) The peculiar velocity field

In the linear regime ( $\frac{\delta\rho}{\rho} \lesssim 1$ ) the local fluctuations in the matter velocity (relative to the Hubble flow) are directly related to the local fluctuations in the mass density by

$$v_k(t) = -i \frac{a(t)}{k} \dot{\delta}(k, t) \quad (8a)$$

or equivalently by,

$$v_k(t) = -i \frac{a(t)}{k} \delta(k, t) H(t) \frac{d \log D(t)}{d \log a(t)}, \quad (8b)$$

where  $v_k(t)$  is the Fourier component of the velocity field and  $H(t)$  is the Hubble parameter. Eqn(8) follows from the Newtonian continuity equation for the NR component or directly from integrating the Newtonian equations of motion, and applies only to scales much smaller than the horizon. These turn out to be the scales of interest, since, as we will discuss below, most of the contribution to the peculiar velocity field comes from scales much smaller than the horizon. For density perturbations of a fixed amplitude  $\delta(k, t)$ , the gravitationally-induced velocities will differ in the different models due to the kinematic factor  $f \equiv \frac{d \log D(t)}{d \log a(t)}$ . For the  $\Omega_{NR} = 1$  model  $D(t) \propto a(t)$  so that  $f = 1$ . For the  $\Lambda \neq 0$ , string network/ $k < 0$ , and fast-moving string models  $f = \Omega_{NR}^{0.57}, \Omega_{NR}^{0.60}$ , and  $\Omega_{NR}^{0.62}$  respectively. For the decaying particle models the functional dependence is more complicated, but  $f$  is still  $< 1$ . For all the smooth component models  $f < 1$ , which reflects the fact that, for a given density fluctuation field, peculiar velocities are more efficiently induced by gravity when there is not a smooth component.

The expected (or *rms*) peculiar velocity (i.e., relative to the CMB) of a spherical volume is obtained by convolving Eqn(8) with an appropriate window function (Kaiser, 1983; Vittorio and Silk, 1985):

$$v_{rms}^2(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 dk |v_k|^2 W^2(kr) \quad (9)$$

The specific form of the window function depends upon how the observed galaxy sample was defined. For simplicity we will assume that the galaxy sample is spherically symmetric, with number density  $n(r') \propto \exp(-r'^2/2r^2)$ . It is easy to show that in this case  $W(kr)^2 = \exp(-k^2r^2)$  (the Fourier transform of a gaussian is a gaussian). If the sample is distributed in a shell, the form of  $W(kr)$  is slightly more complicated, but the exponential form still turns out to be a good approximation for a thick shell of galaxies. It is clear that only perturbations of wavenumber  $k \lesssim r^{-1}$  (wavelengths  $\gtrsim r$ ) contribute significantly to the integral in Eqn(9). For  $r \rightarrow \infty$ , the integral reduces to a simple analytic expression:  $v_{rms}(r) \rightarrow \frac{A^{\frac{1}{2}}}{2\pi} \frac{H_0}{r} \frac{d \log D(t_0)}{d \log a(t_0)}$ .

It must be remembered that the velocities predicted by Eqn(9) measure the peculiar velocity of a randomly placed observer. For inflation-produced fluctuations the primordial density perturbations are gaussian, and so each component of the peculiar velocity is gaussian distributed, and its modulus has a  $\chi^2$ -like distribution with three degrees of freedom. Therefore, the probability of measuring a peculiar velocity in the interval  $v_1 \rightarrow v_2$  is given by:

$$P = \sqrt{\frac{2}{\pi}} \int_{v_1}^{v_2} \left( \frac{v}{v_{rms}} \right)^2 e^{-\frac{1}{2} \left( \frac{v}{v_{rms}} \right)^2} \frac{dv}{v_{rms}} \quad (10)$$

From this it follows that there is a probability of 90% of measuring a velocity  $\frac{1}{1.6} < \frac{v}{v_{rms}} < 3$ .

If the density fluctuations are non-gaussian (as is the case if they are induced by heavy cosmic strings), then Eqn(9) is still formally correct. However, since the expectation value of the peculiar velocity is not gaussian-distributed,  $v_{rms}$  is not a very meaningful quantity as large deviations from it are not exponentially rare.

Throughout, we consider the peculiar velocities on different scales independently. If

correlations between the velocities on different scales are taken into account, the limits presented here can be made even more restrictive (see, Vittorio, Juszkiewicz and Davis, 1986).

*d) Normalization of the primordial spectrum*

At present early Universe physics does not make a definitive prediction as to the overall normalization  $A$ , and  $A$  must be determined from astrophysical/cosmological data. In the stable HDM scenario, we require nonlinearity (defined as the moment at which the *rms* density contrast is 0.6) to occur at a redshift  $z \simeq 3$ . We require this in order to account for the existence of the high redshift quasars.

In the decaying HDM scenario, the reduction of the damping length ensures galaxy formation, and therefore we have normalized the spectrum on scales  $\sim 25h^{-1}Mpc$ , by fitting the observed *rms* fluctuations in the large-scale galaxy counts, as inferred from the  $J_3$  integral over the galaxy-galaxy correlation function  $\xi(r)$   $[\frac{\delta\rho}{\rho}(R)]^2_{rms} = \frac{3}{R^3}J_3$ , where  $J_3 = \int_0^R r^2 dr \xi(r)$ . We could also have normalized the spectrum to give nonlinearity at a specified epoch, e.g.,  $z \simeq z_D$  or  $z \simeq 3$ , as with stable HDM. Both of these procedures lead to a smaller value for the normalization  $A$ , and hence smaller peculiar velocities (by a factor  $\gtrsim 2 - 3$ ). As we shall see, the predicted peculiar velocities for this scenario are already uncomfortably small.

In the case of stable CDM, the absence of any cutoff in the density fluctuation spectrum and its logarithmic divergence on small scales ensure that nonlinearity occurred well in the past on small scales. We normalize the spectrum by requiring the *rms* mass fluctuation averaged over a randomly placed sphere of radius  $r = 8h^{-1}Mpc$  to be unity (Peebles, 1982). This is equivalent to normalizing the spectrum by fitting the variance in the counts

of the bright galaxies. It is clear that we have implicitly assumed that bright galaxies are a good tracer of the overall mass distribution, since we are normalizing to the galaxy distribution. We use the same criterium in the decaying CDM scenario.

#### IV. RESULTS

We will present our predictions in the form of analytical fit to the results obtained by numerically integrating Eqn(9) in different scenarios. All of our fits are accurate to better than 10%.

##### *i) Stable Particles: HDM*

Despite several appealing features, a neutrino-dominated Universe seems to be in conflict with several observations (e.g., see Frenk, Davis, and White, 1983). Kaiser (1983) has pointed out one important problem: in the framework of linear theory a HDM Universe predicts excessive peculiar velocities on scales of  $\sim 25h^{-1}Mpc$ . This apparent conflict involves several assumptions: a) nonlinearity is required at a redshift  $z \sim 3$  (see Sec. IIId); b) a Hubble constant  $h < 0.7$  is assumed in order to have the the age of the Universe  $\gtrsim 10^{10}y$ ; c) the Hart and Davies (1982) data point ( $v_p = 130 \pm 70 km s^{-1}$  on the scale  $\sim 25h^{-1}Mpc$ ) is assumed to be correct.

One wonders if the recent measurement of high peculiar motions on large scales (Collins et al., 1986; Burstein et al., 1986) can resurrect a neutrino-dominated Universe. In a flat HDM dominated Universe we find:

$$v_D = 450h^{-1.37} km s^{-1} \quad (11a)$$

$$v_{25} = 200h^{-1.66} km s^{-1} \quad (11b)$$

$$v_{50} = 100h^{-1.82} km s^{-1} \quad (11c)$$

With the present normalization, at the 90% confidence level one has  $280 < \frac{v_D h^{1.87}}{km s^{-1}} < 1350$  and  $125 < \frac{v_{25} h^{1.66}}{km s^{-1}} < 600$  and  $63 < \frac{v_{50} h^{1.82}}{km s^{-1}} < 300$ . It is clear that one can account for high peculiar velocities on large scales by having  $h \sim 0.5$ . In this case, however, the expected dipole anisotropy far exceeds the observed value. At the 95% confidence level, for  $h = 0.5$ ,  $v_D > 723 km s^{-1}$ , while the observed "corrected" dipole anisotropy is  $\sim 500 km s^{-1}$ .

ii) *Stable Particles: CDM*

This scenario has been widely studied in the past years both in the linear and in the nonlinear regime. If the primordial fluctuations are adiabatic the predicted peculiar velocity field in the  $k \leq 0$  case is found to be:

$$v_D = 322 km s^{-1} \Omega_{NR}^{+0.03} h^{-0.57} \quad (12a)$$

$$v_{25} = 156 km s^{-1} \Omega_{NR}^{-0.18} h^{-0.78} \quad (12b)$$

$$v_{50} = 83 km s^{-1} \Omega_{NR}^{-0.33} h^{-0.92} \quad (12c)$$

$v_D$  is quite insensitive to  $\Omega_{NR}$ , while both  $v_{25}$  and  $v_{50}$  depend inversely on  $\Omega_{NR}$ . This dependence on  $\Omega_{NR}$  arises because in an open Universe  $\frac{d \log D(t_0)}{d \log a(t_0)} \simeq \Omega_{NR}^{0.6}$  (Peebles, 1980) and  $\delta_k$  itself depends on  $\Omega_{NR}$  (cf., Eq.(6)). Due to the asymptotic behaviour of the CDM density fluctuation spectrum, the contribution to the integral in Eq.(9) per unit logarithmic interval of  $k$  is proportional to  $k^2 (k \rightarrow 0)$  and to  $k^{-2} (k \rightarrow \infty)$ , and has a maximum at  $k \sim \frac{2\pi}{L_{EQ}}$ . This justifies the Newtonian approach described in Sec. IIIc and shows that for a fixed normalization, as long as  $L_{EQ} \gtrsim 10 h^{-1} Mpc$  the predicted dipole anisotropy should be approximately independent of  $\Omega_{NR}$ . Now consider the limit  $r \rightarrow \infty$ . Utilizing the asymptotic formulae, it is easy to see that the dependence on the density parameter should approach  $\Omega_{NR}^{-0.4}$ : while the velocities are reduced by a factor  $\Omega_{NR}^{0.6}$ , the amplitude of

fluctuations on large scales for a fixed normalization is increased by about a factor  $\Omega_{NR}^{-1}$ . On the intermediate scales, we expect an intermediate dependence upon  $\Omega_{NR}$ .

The introduction of a cosmological constant ( $\Omega_\Lambda = 1 - \Omega_{NR}$ ), light, fast-moving strings ( $\Omega_{FS} = 1 - \Omega_{NR}$ ), or a string network ( $\Omega_{NET} = 1 - \Omega_{NR}$ ) to save the flat model does not change substantially the peculiar velocity predictions. Since  $k_\nu$  in Eqn(5) and  $\alpha, \beta, \gamma$  in Eqns(6) and (7) depend upon  $L_{EQ}$  which is  $\propto \Omega_{NR}^{-1}$ , the shape of the density fluctuation spectrum does not change, and the factor  $\frac{d \log D(t)}{d \log a(t)}$  is very similar for these three models ( $= \Omega_{NR}^{0.57}, \Omega_{NR}^{0.62}, \Omega_{NR}^{0.60}$  respectively), despite the fact that the growth of fluctuations in the  $\Lambda$  model is far more efficient than in the other three cases (Charlton and Turner, 1986).

One way of reconciling the theoretical prejudice for an  $\Omega_{NR} = 1$  Universe with the observed low values of  $\Omega_{NR}$  is to assume that galaxies are a biased tracer of the overall mass distribution (see, e.g., Bardeen et al., 1986). In this scenario galaxies form only in the highest peaks ( $\nu \equiv \frac{\delta}{\delta_{rms}} \gg 1$ ) of the density field: this would imply that the overall mass distribution is more uniform and that the amplitude of the *rms* density fluctuations is, for a fixed normalization criterium, reduced by a factor  $\nu^{-1}$ . It has been argued (Vittorio and Silk, 1985a) that in an  $\Omega_{NR} = 1$  CDM dominated Universe the biased galaxy formation scenario may predict peculiar velocities that are too low. Peacock and Heavens (1985) suggest that one is forced, in any case, to have biased galaxy formation since most of the maxima of the density field, which are plausible seeds of galaxy formation, have a density contrast  $\delta \sim 2\delta_{rms}$ : this would imply  $\nu = 2$ .

With the present normalization, at the 90% confidence level one has  $200 < \frac{v_D h^{0.57} \nu}{km s^{-1}} < 970$  and  $100 < \frac{v_{25} \Omega_{NR}^{0.15} h^{0.78} \nu}{km s^{-1}} < 470$  and  $50 < \frac{v_{50} \Omega_{NR}^{0.35} h^{0.92} \nu}{km s^{-1}} < 250$ . It is clear that for  $\nu \sim 3$  the predictions of an  $\Omega_{NR} = 1$  CDM scenario are far too small to account for the

high peculiar motions reported on scales  $\sim 50h^{-1}Mpc$ .

Eq.(12) suggests that one can have high peculiar velocities with low values both of the density parameter  $\Omega_{NR}$  and of the Hubble constant. However the upper limit to the small-scale anisotropy of the CMB (Uson and Wilkinson, 1984) implies a lower limit on the density parameter  $\Omega_{NR}$ , since in an  $\Omega_{NR}$  Universe the growth of fluctuations is inhibited (Charlton and Turner, 1986). For a CDM dominated Universe with  $k < 0$  (or with  $\Omega_{NET} = 1 - \Omega_{NR}$ , since  $\rho_{NET} \propto a^{-2}$ ) the small-scale isotropy implies:  $\Omega_{NR}h\nu > 0.2$  (Vittorio and Silk, 1984; Bond and Efstathiou, 1984). The case of light, fast-moving strings has not been studied yet, although the bound should be more restrictive since perturbations undergo less growth than in an open Universe (Turner, 1985a). With a non-vanishing cosmological constant the growth of density fluctuation is more efficient, although never as efficient as in a  $\Omega_{NR} = 1$  Universe, and the CMB constraint is:  $\Omega_{NR}h\nu > 0.05$  (Vittorio and Silk, 1985c). In sum, it is possible that a low density CDM dominated Universe may provide a way of having high peculiar velocities on large scales while still being consistent with the present upper limit to the CMB small-scale isotropy, especially if  $\Lambda = 1 - \Omega_0$ .

If the primordial fluctuations are of the isocurvature type, then the peculiar velocity field for the  $k < 0$  model is:

$$v_D = 522 km s^{-1} \Omega_{NR}^{-0.05} h^{-0.64} \quad (13a)$$

$$v_{25} = 304 km s^{-1} \Omega_{NR}^{-0.22} h^{-0.83} \quad (13b)$$

$$v_{50} = 178 km s^{-1} \Omega_{NR}^{-0.38} h^{-0.99} \quad (13c)$$

At the 90% confidence level one has  $326 < \frac{v_D h^{0.64} \nu}{km s^{-1}} < 1566$ ,  $190 < \frac{v_{25} \Omega_{NR}^{0.22} h^{0.83} \nu}{km s^{-1}} < 912$  and  $111 < \frac{v_{50} \Omega_{NR}^{0.38} h^{0.99} \nu}{km s^{-1}} < 534$ .

The values predicted in this case are higher than in the adiabatic case. This is due to the fact that the isocurvature fluctuation spectrum is flatter on small-scales than the adiabatic spectrum. Thus, for a fixed normalization, the density fluctuations on large scales are larger, and because of this the predicted velocity field has a higher amplitude. The scaling with  $\Omega_{NR}$  is similar to the adiabatic case, since the limiting behaviour of the two spectra are similar.

For  $h \sim 0.5$ ,  $\nu \sim 2$  and  $\Omega_{NR} \sim 0.2$  this scenario seems to provide good agreement with the present observational data. However, it is ruled out by the excessive large-scale CMB anisotropy which it predicts (Efstathiou and Bond, 1986).

### iii) *Unstable particles: HDM*

As discussed in Sec. I, the decaying particle scenario is also able to reconcile a flat Universe with the observed low values of  $\Omega_{NR}$ . Since the damping length decreases inversely with the decay redshift, one might also hope to resolve some of the other difficulties associated with the stable neutrino scenario (Turner et al., 1984). High redshifts of decay result in a very small damping length ( $\lesssim 10h^{-1}Mpc$ ), which implies that the density fluctuation spectrum is basically the primordial one ( $\propto k$ ) on the scales of interest. This spectrum fails in reproducing the large-scale structure. The normalization chosen, the  $J_3$  integral, minimizes this difficulty. The peculiar velocity field is in this case:

$$v_D = 500(1 + z_D)^{-0.74} km s^{-1} \quad (14a)$$

$$v_{25} = 270(1 + z_D)^{-0.85} km s^{-1} \quad (14b)$$

$$v_{50} = 150(1 + z_D)^{-0.88} km s^{-1} \quad (14c)$$

The values given in Eq.(14) are an analytical fit which is good for  $3 \lesssim 1 + z_D \lesssim 10$ . At the 90% confidence level one has  $312 < \frac{v_D(1+z_D)^{0.74}}{km s^{-1}} < 1500$ ,  $169 < \frac{v_{25}(1+z_D)^{0.85}}{km s^{-1}} < 810$ ,

and  $94 < \frac{v_{50}(1+z_D)^{0.88}}{km\ s^{-1}} < 450$ . [Here we have considered a Universe with  $\Omega_{NR} = 0.2$  and  $h = 0.5$ —remember, a flat Universe dominated by relativistic particles is very youthful,  $H_0 t_0 \simeq 0.55$ . In addition, for  $\Omega_{NR} = 0.2$ ,  $z_D$  must be  $\gtrsim 2$  in order that the unstable particles have sufficient time to decay.]

Confirmation of peculiar velocities  $\sim 600 km s^{-1}$  on large scale ( $\gtrsim 50 h^{-1} Mpc$ ) will constitute a serious difficulty for this scenario since for  $z_D \gtrsim 2$ , at the 95% confidence level  $v_{50} < 170 km s^{-1}$ . Recall that if the spectrum of perturbations is normalized by the epoch of nonlinearity the predicted velocities will be even smaller. It should also be remembered that this scenario suffers from other difficulties, the most serious of which seems to be the requirement of an high redshift of decay ( $1 + z_D \gtrsim 5 - 10$ ) in order to be consistent with our Virgocentric infall (Efstathiou, 1985; Hoffman, 1986). This seems to be in mild conflict with the limit of  $(1 + z_D) \lesssim 5$  which is imposed by the small-scale isotropy of the CMB (Vittorio and Silk, 1985c).

*iv) Unstable Particles: CDM*

Next we consider the peculiar velocity field in the case of unstable CDM. For adiabatic fluctuations we find:

$$v_D = 684(1 + z_D)^{-1.70} km s^{-1} \quad (15a)$$

$$v_{25} = 342(1 + z_D)^{-1.78} km s^{-1} \quad (15b)$$

$$v_{50} = 181(1 + z_D)^{-1.80} km s^{-1} \quad (15c)$$

At the 90% confidence level one has  $430 < \frac{v_D(1+z_D)^{1.70}}{km\ s^{-1}} < 2050$  and  $210 < \frac{v_{25}(1+z_D)^{1.78}}{km\ s^{-1}} < 1025$  and  $115 < \frac{v_{50}(1+z_D)^{1.80}}{km\ s^{-1}} < 550$

For decaying CDM and isocurvature fluctuations we find:

$$v_D = 836(1 + z_D)^{-0.83} km s^{-1} \quad (16a)$$

$$v_{25} = 566(1 + z_D)^{-1.01} \text{ km s}^{-1} \quad (16b)$$

$$v_{50} = 346(1 + z_D)^{-1.08} \text{ km s}^{-1} \quad (16c)$$

At the 90% confidence level one has  $522 < \frac{v_D (1+z_D)^{0.88}}{\text{km s}^{-1}} < 2500$  and  $354 < \frac{v_{25} (1+z_D)^{1.01}}{\text{km s}^{-1}} < 1698$  and  $215 < \frac{v_{50} (1+z_D)^{1.08}}{\text{km s}^{-1}} < 1038$ .

We again choose  $\Omega_{NR} = 0.2$  and  $h = 0.5$ ; as before, the values in Eqn(15) and Eqn(16) are fits to numerical results, for  $3 \lesssim 1 + z_D \lesssim 10$ . Irrespective of the nature of the initial fluctuations (adiabatic or isocurvature), a CDM decaying scenario is unable to explain the reported high peculiar velocities on large scales ( $\sim 50h^{-1} \text{ Mpc}$ ). Also, if density fluctuations are adiabatic, the present upper limit on the CMB small scale anisotropy implies  $z_D \lesssim 5 - 10$  (Kolb, Olive, Vittorio, 1986; also, see Turner, 1985c). In addition, the Virgocentric infall argument applies to CDM as well. Moreover, a CDM decaying scenario seems unable to account for flat galactic rotation curves (Flores et al., 1986).

## V. Summary

We have examined the expected peculiar velocity field in 2-component, flat models of the Universe. Our main conclusion are:

1) A flat HDM dominated Universe does not seem to be resurrected by the recent evidence of large peculiar motions on large scales ( $\gtrsim 50h^{-1} \text{ Mpc}$ )

2) A flat CDM dominated Universe with adiabatic density fluctuations produces a peculiar velocity field in reasonable agreement with observations on intermediate scales ( $\lesssim 30h^{-1} \text{ Mpc}$ ), but it has difficulties in reproducing large scale ( $\gtrsim 50h^{-1} \text{ Mpc}$ ), large amplitude bulk motions. If galaxies do not trace the overall mass distribution (as in the biased scenarios), then the velocity field has a hopelessly small amplitude. With

isocurvature fluctuations, a CDM-dominated Universe has large peculiar velocities, but unfortunately predicts an excessive large-scale CMB anisotropy.

3) A flat Universe dominated by the relativistic decay products of an unstable relic predicts small amplitude, large scale peculiar velocity field. The evidence for large-scale, large-amplitude peculiar velocities, if confirmed, could constitute yet another problem for this scenario.

4) A low-density, CDM-dominated Universe with adiabatic fluctuations may provide a model which is consistent with the recent claims of large-scale, large-amplitude peculiar velocities. Unless there is a smooth component to the mass density (e.g.,  $\Lambda \neq 0$ , or light strings), one would have to abandon theoretical prejudices for a flat Universe to embrace such a model. Low-density, CDM-dominated models (with or without a smooth component) clearly deserve further attention.

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